

# Space-Time Uncertainty and Approaches to D-Brane Field Theory

Tamiaki YONEYA

*Institute of Physics, University of Tokyo  
Komaba, Meguro-ku, Tokyo 153-8902*

In connection with the space-time uncertainty principle which gives a simple qualitative characterization of non-local or non-commutative nature of short-distance space-time structure in string theory, author's recent approaches toward field theories for D-branes are briefly outlined, putting emphasis on some key ideas lying in the background. The final section of the present report is devoted partially to a tribute to Yukawa on the occasion of the centennial of his birth.

## §1. Introduction

Most of physicists who are exploring possible roads towards the unification of General Relativity and Quantum Theory would agree that the space-time structure at short distances near the Planck scale can never be fully understood in terms of traditional Riemannian geometry. Apparently, various geometrical notions in Riemannian geometry become more and more inappropriate in describing quantum theory as we try to probe shorter and shorter distances. One of the concrete phenomena exhibiting this is of course the non-renormalizability of ultraviolet divergences of general relativistic field theories, including supergravity which may be regarded as the final and most extreme outcome of unified field theories within the framework of *classical* local field theory. One can imagine a multitude of *ad hoc* mechanisms which might resolve the ultraviolet difficulty, such as space-time lattices, non-local field theories, space-time quantizations, so on and so forth. However, most of such attempts in the past except for string theory lead either to inconsistent theories which cannot be compatible with unitarity of quantum theory, or to uncontrollable theories without reliable principles to fix their contents. At least in its known perturbative definitions, string theory is quite rigid and controllable, and its S-matrix is perfectly consistent with general principles of unitarity.

We should expect that the right approach toward our goal would necessarily be a unified theory of all interactions including gravity. It is remarkable that string theory has been providing an ideal perspective on this fundamental question, even though the string theory stemmed originally from what seemed nothing to do with gravity and unification of forces, and is admittedly still at an stage of an incomplete theory for making definite observable predictions. Its impressive developments of more than three decades convince us of the belief that the string theory is exhibiting some of key ingredients for achieving the ultimate unification. It is the author's conviction that the string theory is far deeper than what has been understood so far, and that there must be many facets yet to be discovered. This was basically the author's attitude when he proposed the space-time uncertainty relation as a simple

qualitative characterization of the short-distance space-time structure of the string theory, 20 years ago.

In the present talk, we would like to first revisit the space-time uncertainty relation. Then starting from it as a motivation, I will go into some of my recent attempts toward the quantum field theory for D-branes. It turned out that our models for field theory for D-branes, in particular D-particles, have certain features which are not unrelated to the old idea of Yukawa, especially, the idea of ‘elementary domains’. In fact, the idea of space-time uncertainty itself has some flavor related to this idea. So in the final part of the present talk, I would like to devote my discussion partially to a homage to Yukawa, to an extent that is related to our formulation of D-brane field theory. That seems appropriate in this Nishinomiya-Yukawa symposium, especially since this year 2007 is the centennial of his birth.

## §2. Revisiting the space-time uncertainty principle

Let me allow to quote some sentences from my talk<sup>1)</sup> at the 2nd Nishinomiya-Yukawa symposium in 1987, in which I have given one of the earliest accounts on the idea<sup>2)3)</sup> of the space-time uncertainty relation.

*“This implies that the very notion of string itself must be a more fundamental geometrical entity encompassing the notion of metric tensor, connection and curvature and so forth, of Riemannian geometry which is the appropriate language for General Relativity based on local field theories. Note that the string theory by definition assumes that everything is made out of strings. Even the geometry of space-time must be expressed by using only the notion of strings. This seems to require a fundamental revision on the usual concept of the space-time continuum. Then the constant  $\alpha'$  should be interpreted as putting certain limitation on the usual concept of Riemannian geometry, just as the Planck constant  $h$  puts a limitation on the classical concept of the phase space. From this point of view, we expect some clear characteristic relation expressing the limitation, in analogy with the commutation relation or Heisenberg’s uncertainty principle in quantum theory.*

*I now would like to suggest a possible hint along this line of arguments. Namely, a possible expression of duality in space-time languages could be*

$$\Delta t \Delta \ell \gtrsim 4\pi\alpha' \quad (2.1)$$

*where  $\Delta t$  is an appropriate measure of the indeterminacy of propagation length of a string state and the  $\Delta \ell$  is an appropriate measure of the intrinsic extendedness of a propagating string. This “space-time indeterminacy relation” means that any observation in string theory probing short “time” ( $\Delta t \rightarrow 0$ ) or small “distance” ( $\Delta \ell \rightarrow 0$ ) structure in space-time is associated with large indeterminacy in the “dual” variable,  $\Delta \ell$  or  $\Delta t$ , respectively. Thus (2.1) sets a limitation about the smallness of the space-time domain where arbitrary possible observation in string theory is performed.*

*In the limit of small  $\Delta t$ , (2.1) can in fact be derived<sup>2)</sup> by re-interpreting the Heisenberg relation  $\Delta t \Delta E \gtrsim h$ . ....”*

The last statement came from the recognition that the typical intrinsic length scale

$\ell$  of a string with large energy  $E$  is  $\ell \sim \alpha' E$  (using the natural unit  $c = 1 = \hbar$  here and in what follows for simplicity). Namely, because of the huge degeneracy of string states a large energy given to a string by interaction are expensed dominantly in virtual intermediate states by exciting higher string modes, rather than by boosting with a large center-of-mass momentum, within the constraint of momentum conservation.

Here I only summarize its meanings in connection with recent development of string theory related to D-branes. For more detailed discussions, I would like to refer the reader to the works mentioned above and also to ref.<sup>4)</sup> which includes expository accounts from various relevant viewpoints.

- (1) The relation (2.1) gives a characterization of world-sheet (*i. e.* open-closed and channel) duality in terms of simple space-time language. It can also be regarded as a consequence of conformal symmetry, especially of modular invariance. As such it has a universal applicability at least qualitatively in understanding string dynamics including D-branes, providing that due care is paid for its application since precise definition of the uncertainties  $\Delta t, \Delta \ell$  has not been given, unfortunately at the present stage of development.
- (2) In connection with this, I emphasize that the traditional notion of “minimal length”  $\ell \gtrsim \ell_s \sim \sqrt{\alpha'}$  is too restrictive to characterize the dynamics of D-branes. For instance, in the case of D-particle, typical scales<sup>5)</sup> are

$$\Delta \ell \sim g_s^{1/3} \ell_s, \quad \Delta t \sim g_s^{-1/3} \ell_s \quad (2.2)$$

in the weak-coupling regime  $g_s \ll 1$ . The spatial scale  $g_s^{1/3} \ell_s$  is much smaller than the typical string scale  $\ell_s$  while the time scale is conversely larger, in conformity with the space-time uncertainty relation (2.1).

- (3) The relation is consistent with the Yang-Mills description of low-energy effective dynamics of D-branes. For instance, the case of D0-branes, D-particles, is described by the 1-dimensional action,

$$S_{SYM} = \int dt \text{Tr} \left( \frac{1}{2g_s \ell_s} D_t X^i D_t X^i + i \theta D_t \theta + \frac{1}{4g_s \ell_s^5} [X^i, X^j]^2 - \frac{1}{\ell_s^2} \theta^T \Gamma^i [\theta, X^i] \right). \quad (2.3)$$

This action has a scaling symmetry<sup>6)</sup> under

$$X^i \rightarrow \lambda X^i, \quad t \rightarrow \lambda^{-1} t, \quad g_s \rightarrow \lambda^{3-p} g_s \quad (2.4)$$

which directly gives (2.2). This symmetry, together with the susy condition  $\lim_{v \rightarrow 0} S_{eff} = 0$ , constrains the form of 2-body effective action in the form

$$S_{eff} = \int dt \left( \frac{1}{2g_s \ell_s} v^2 - \sum_{k=0}^{\infty} c_k \frac{v^{2k} \ell_s^{4k-2}}{r^{4k-1}} + O(g_s) \right) \quad (2.5)$$

and hence effectively governs some of important gross features of D-particle scattering. If we re-scale the unit globally, the transformation (2.4) is equivalent to the light-cone scaling symmetry in the context of the so-called DLCQ M(atrrix)-theory<sup>7)</sup>  $t \sim x^+ \rightarrow \lambda^{-2} x^+$ ,  $X^i \rightarrow X^i$ ,  $x^- \sim R_{11} (\equiv g_s \ell_s) \rightarrow \lambda^2 R_{11}$ .

An important question is what the proper mathematical formulation of the space-time uncertainty relation should be. Most of discussions at early stages have been within the framework of perturbation theory based on the world-sheet picture for strings. An obvious possibility which does not directly rely on the world-sheet picture would be to assume some non-commutative structure for the space-time coordinates. One such example is to start from an algebra like  $[X^\mu, X^\nu]^2 \sim \ell_s^4$  leading us to an infinite-dimensional matrix model which is quite akin to the so-called type IIB (IKKT) matrix model, as discussed in detail in ref.<sup>9)</sup> However, in such approaches, meanings, both physically and mathematically, of the space-time coordinate ‘operators’  $X^\mu$  as matrices are quite obscure: What are they coordinates of? Are they D-instantons? But then how are they related to physical observables? Or are they space-time *itself*, as for instance in the sense that has been argued in<sup>10)</sup>? .... In any case, the space-time uncertainty relation in string theory cannot be the primordial principle and should be regarded at best as a rough and qualitative consequence resulting from some unknown but deeper principles which govern the theory.

Here I would like to recall that the success of quantum field theory in the previous century taught us that the notion of fields is more important than the coordinates, in dealing with various physical phenomena. When we make any physical observations, we do not measure the coordinates of physical objects directly, but rather effects or events caused by the objects. The geometry of space-time must then be expressed in terms of fields defined on space-time coordinates which themselves have no observational meanings directly other than as labels for events. Of course, General Relativity itself where every physical observables can be expressed in terms of fields is a realization of this general idea. String field theory is also an attempt to realize this<sup>\*)</sup> in string theory. Moreover, the notion of quantized fields gave above all the final reconciliation between wave and particle, the primordial duality lying in the foundation of modern quantum physics.

In string theory we now have understood through developments during recent 10 years that D-branes<sup>14)</sup> can be regarded as more basic elements of string theory, rather than the (‘fundamental’) strings. One question arising then is what are D-branes from this viewpoint. Are they particle-like or wave-like objects? All previous discussions of D-branes are actually based on the former particle-like view on D-branes: if we go back to the effective Yang-Mills descriptions such as (2·3), they are obviously configuration-space formulations, in the sense that we deal directly with the coordinate matrices  $X^\mu$  whose diagonal components are nothing but the transverse positions of D-branes. The Yang-Mills description is an approximation ignoring massive open-string degrees of freedom associated to D-branes. Thus, open-string field theories with Chan-Paton gauge symmetry are still 1st-quantized formulations of D-branes, even though they are 2nd-quantized for open strings. Open-string fields are in this sense nothing more than the collective coordinates for D-branes. To my knowledge, truly 2nd-quantized theory of D-branes has never been discussed in the

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<sup>\*)</sup> It is well known that a similar viewpoint, ‘pointless geometry’, has been emphasized in.<sup>11)</sup> It might also be useful here to remind that the original conjecture<sup>12)</sup> by the present author of purely cubic actions<sup>13)</sup> for string field theory was motivated by this line of thought.

literature. I shall explain mainly my recent attempts towards field theory of D-branes, and argue that successful D-brane field theories would probably provide us a new perspective on the duality between open and closed strings, which is a basic principle in the heart of various unifications achieved by string theory.

### §3. Why D-brane field theory?

Even apart from the above line of thought, whether *field* theory for D-branes is possible or not is an interesting question *per se*. One motivation for D-brane field theory can be explained by making an analogy with the well-known Coleman-Mandelstam duality<sup>15)</sup> between massive Thirring model and sine-Gordon model in two-dimensional field theory. The kink-like soliton excitations in the latter can be described in the former as elementary excitations corresponding to Dirac fields. This can be established explicitly by constructing operators which create and annihilate the kink solitons in the latter. D-branes might be understood as analog of the kinks of the sine-Gordon model, since D-branes in general appear as non-trivial classical solutions (with or without sources) in the low-energy supergravity approximation to closed-string field theories. For string theory, the Schwinger model also provides an interesting analogy with open/closed string duality: the one-loop of massless Dirac fields gives a pole singularity corresponding to massive scalar excitation. The massive scalar (or sine-Gordon) field is the analog of closed strings, while the massless Dirac (or massive Thirring) field is that of open strings.

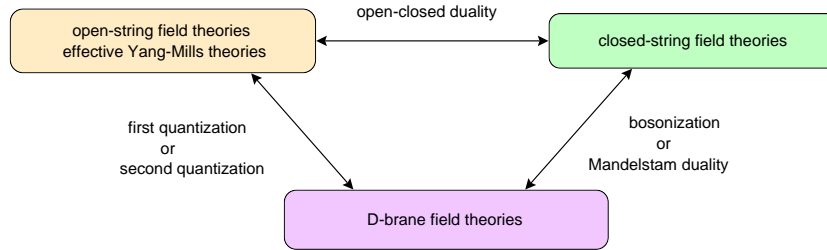
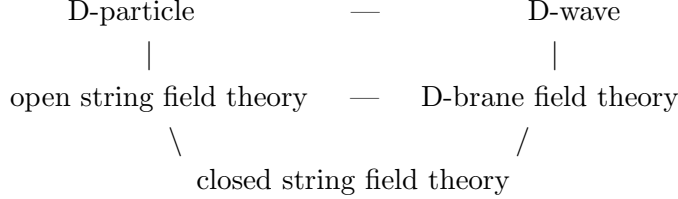


Fig. 1. A trinity of dualities: D-brane field theories point toward the third possible formulation of string theory treating D-branes as elementary excitations.

These analogies suggest the possibility of some field theory for D-branes which is dual to closed-string field theory, in a similar way as the massive Thirring model is dual to the sine-Gordon model. The field theory of D-branes must give a 2nd quantized formulation of multi-body D-brane systems which in the non-relativistic low-energy approximation should be equivalent to the Yang-Mills theory of D-branes. Hence, we should be able to find some framework for 2nd-quantizing the Yang-Mills theory in the sense of D-branes. Namely, the number of D-branes corresponds to the size of the Yang-Mills gauge groups. Therefore, D-brane field operators must act as operators which change the size of the gauge groups in the Fock space of D-branes.

The idea can be summarized by the diagram in Fig. 1. We expect the existence of third possible formulation of string (and/or M) theory. D-brane field theories would

give a new bridge which provides an explanation of open/closed string duality as a special case of the general notion of particle/wave duality. Namely, open string field as a particle picture for D-branes is dual to D-brane field theory, a wave picture, by 2nd-quantization, and then D-brane field theory is dual to closed-string field theory by a generalized bosonization, as summarized by the following diagram:



#### §4. Gauge symmetry as a quantum-statistics symmetry

In attempting the 2nd-quantization of D-brane quantum mechanics, one of the most interesting problems is that the permutation symmetry which is the basic quantum-statistical symmetry governing the ordinary 2nd-quantization of many-particle systems must be replaced by the continuous gauge group  $U(N)$ . Let  $X_{ab}^i$  ( $a, b, \dots = 1, 2, \dots, N$ ) be the coordinate matrices of  $N$  D-branes with  $i$  being the transverse spatial directions. Then, the gauge symmetry transformation has an adjoint action as

$$X_{ab}^i \rightarrow (U X^i U^{-1})_{ab}, \quad U \in U(N). \quad (4.1)$$

The diagonal components  $X_{aa}^i$  represent the positions of  $N$  D-branes, while the off-diagonal components correspond to the lowest degrees of freedom of open strings connecting them. If we constrain the system such that only the diagonal elements survive, the gauge symmetry is reduced to permutations of diagonal elements. It is therefore clear that one of crucial ingredients in trying 2nd-quantizing the system is how to deal with this feature. In general, D-branes can be treated neither as ordinary bosons nor fermions.

There is, however, one exception where we can apply the usual 2nd-quantization for matrix system. If there is only one matrix degree of freedom as in the case of the old  $c = 1$  matrix model, we can diagonalize the matrix coordinate from the outset, and the Hilbert space is then replaced by that of  $N$  free fermions, moving in a fixed external potential, whose coordinates are those eigenvalues. The 2nd-quantization is then reduced to that of usual non-relativistic fermions. Unfortunately, this method cannot be extended to cases where we have to deal with many matrices. One of popular methods in the past has been to try to represent such systems by a set of gauge invariants. A typical idea along this line has been to use Wilson-loop like operators. However, from the viewpoint of formulating precise dynamical theory, putting aside its use merely as convenient observables to probe various physical properties, use of Wilson loops or similar set of invariants is quite problematic. The reason is that it is usually extremely difficult to separate independent degrees of freedom out of the set of infinite number of invariants. This difficulty makes almost hopeless any attempt of 2nd-quantization for matrix systems in our sense.

I shall suggest an entirely new way of dealing with the situation, by restricting ourselves to the system of D0-branes in the Yang-Mills approximation. Before proceeding to this, I wish to mention another related work<sup>16)</sup> of myself, in which an attempt extending the method of the  $c = 1$  matrix model was made for a description of the extremal correlators for a general set of 1/2-BPS operators for D3-branes. This case is very special in that we can justifiably use a free-field approximation.<sup>17)</sup> It was shown that in this approximation the extremal correlators can be expressed in terms of bilinear operators of D3-brane fields. The normal modes of the D3-brane fields are composite operators of the form  $(b_n c_I, b_n^\dagger c_I^\dagger)$ , where  $b_n, b_n^\dagger$  carrying only energy index  $n$  are usual fermion mode operators obeying the standard canonical anti-commutation relations, while  $c_I, c_I^\dagger$  obey the Cuntz algebra<sup>18)</sup> in the form

$$c_{I_1} c_{I_2}^\dagger = \delta_{I_1 I_2}, \quad \sum_{I=0}^{\infty} c_I^\dagger c_I = 1$$

and carry only internal R-symmetry indices  $I$  which denote an appropriate basis for the completely symmetric and traceless representations of  $SO(6)$ . As such, the D3-brane fields do not satisfy any simple commutation relations. Yet the bilinears representing the ordinary gauge invariants and acting on physically allowed states satisfy the standard (commutative) algebra

A lesson from this work was that D-brane fields cannot be described in any existing canonical framework of quantum field theory. It seems to be inevitable that some drastic extension of the general framework of quantum field theory is necessitated in order to carry out the 2nd-quantization of D-branes in general. We have to invent a new mathematical framework for establishing truly 2nd-quantized field theories of D-branes, which obey *continuous* quantum statistical symmetry. This is an entirely new way of counting the degrees of freedoms of physical excitations. The ‘D-brane statistics’ is far more alien than Bose-Einstein or Fermi-Dirac statistics were.

## §5. An attempt toward D-particle field theory

I now describe my attempt toward a quantum field theory of D-particles, in the non-relativistic Yang-Mills approximation. What is to be done is to find an appropriate mathematical structure for dealing with the Fock space of Yang-Mills quantum mechanics in terms of field operators which change the number of D-particles. For each  $N$ , the configuration space for the wave function consists of the  $N \times N$  matrix coordinate  $X_{ab}$ . In the present report we only explain some of basic ideas, by referring more details to the original work.<sup>19)</sup> Here and in what follows, we will suppress spatial indices and also Grassmannian degrees of freedom.

Let us recall the usual 2nd-quantization for ordinary bosons. The configuration-space wave functions of  $N$  particles are replaced by a state vector  $|\Psi\rangle$  in a Fock space  $\mathcal{F} = \sum_N \bigoplus \mathcal{H}_N$  as

$$\begin{aligned} & \Psi(x_1, x_2, \dots, x_N) \\ \Rightarrow |\Psi\rangle &= \left( \prod_{i=1}^N \int d^d x_i \right) \Psi(x_1, x_2, \dots, x_N) \psi^\dagger(x_N) \psi^\dagger(x_{N-1}) \cdots \psi^\dagger(x_1) |0\rangle \end{aligned} \quad (5.1)$$

where the field operators define mappings between  $\mathcal{H}_N$ 's with different  $N$ ,

$$\psi^\dagger(x) : \mathcal{H}_N \rightarrow \mathcal{H}_{N+1},$$

$$\psi(x) : \mathcal{H}_N \rightarrow \mathcal{H}_{N-1}.$$

The quantum-statistical symmetry is expressed as

$$\psi^\dagger(x_N)\psi^\dagger(x_{N-1})\cdots\psi^\dagger(x_1)|0\rangle = \psi^\dagger(x_{P(N)})\psi^\dagger(x_{P(N-1)})\cdots\psi^\dagger(x_{P(1)})|0\rangle \quad (5.2)$$

and

$$\psi(y)\psi^\dagger(x_N)\cdots\psi^\dagger(x_1)|0\rangle = \frac{1}{(N-1)!} \sum_P \delta^d(y-x_{P(N)})\psi^\dagger(x_{P(N-1)})\cdots\psi^\dagger(x_{P(1)})|0\rangle \quad (5.3)$$

where the summation is over all different permutations  $P : (12\dots N) \rightarrow (i_1 i_2 \dots i_N)$ ,  $P(k) = i_k$ . Of course, the whole structure is reformulated as a representation theory of the canonical commutation relations of the field operators acting on the Fock vacuum  $|0\rangle : \psi(x)|0\rangle = 0$ ,

$$[\psi(x), \psi^\dagger(y)] = \delta^d(x-y), \quad [\psi(x), \psi(y)] = 0 = [\psi^\dagger(x), \psi^\dagger(y)].$$

In particular, the last two commutators represent the permutation symmetry, or Bose statistics.

If we compare this structure with the Fock space of D-particles which consists of the Yang-Mills theory of different  $N$ , two crucial features are that

- (a) The increase of the degrees of freedom in the mapping  $\mathcal{H}_N \rightarrow \mathcal{H}_{N+1}$  is  $d_N \equiv d(2N+1) = d((N+1)^2 - N^2)$ , instead of  $d = d(N+1) - dN$  which is independent of  $N$ .
- (b) The statistical symmetry is a continuous group, the adjoint representation of  $U(N)$ , instead of the discrete group of permutations  $\{P\}$ .

The feature (a) indicates that D-particle fields which we denote by  $\phi^\pm[z, \bar{z}; t]$ , creating or annihilating a D-particle, must be defined on a base space with an infinite number of coordinate components, since  $d_N \rightarrow \infty$  as  $N \rightarrow \infty$ . But, if they act on a state with a definite number  $N$  of D-particles, only the finite number,  $d_N$ , of them must be activated, and the remaining ones should be treated as dummy variables. In terms of the matrix coordinates, we first redefine components of these infinite dimensional space as

$$z_a^{(b)} = X_{ab} = \bar{X}_{ba} \quad \text{for } b \geq a, \quad (5.4)$$

which is to be interpreted as the  $a$ -th component of the (complex) coordinates of the  $b$ -th D-particle. The assumption here is that the field algebra and its representation should be set up such that we can effectively ignore the components  $z_a^{(b)}, \bar{z}_a^{(b)}$  with  $a > b$  for the  $b$ -th operation in adding D-particles. Hence, the matrix variables are embedded into sets of the arrays of infinite-dimensional complex vectors ( $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2, \dots$ ). Note that the upper indices with braces discriminate the D-particles by ordering them, whereas the lower indices without brace represent the



components of the infinite dimensional coordinate vector  $(z, \bar{z}) = \{z_1, \bar{z}_1, z_2, \bar{z}_2, \dots\}$  for each D-particle.

Thus we define creation,  $\phi^+[z, \bar{z}]$ , and annihilation,  $\phi^-[z, \bar{z}]$ , operators on the base space of an infinite-dimensional vector space consisting of  $(z_n, \bar{z}_n)$  with  $n = 1, 2, 3, \dots$ . The process of creating and annihilating a D-particle must be defined conceptually (time being suppressed) as

$$\begin{aligned} \phi^+ : |0\rangle &\rightarrow \phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle \rightarrow \phi^+[z^{(2)}, \bar{z}^{(2)}]\phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle \rightarrow \dots, \\ \phi^- : 0 &\leftarrow |0\rangle \leftarrow \phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle \leftarrow \phi^+[z^{(2)}, \bar{z}^{(2)}]\phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle \leftarrow \dots. \end{aligned}$$

Pictorially this is illustrated in Fig. 2.

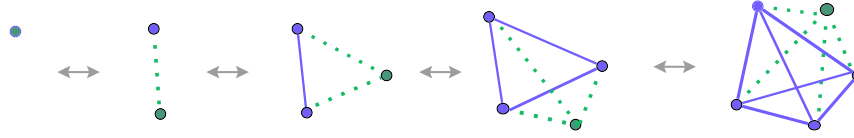


Fig. 2. The D-particle coordinates and the open strings mediating them are denoted by blobs and lines connecting them, respectively. The real lines are open-string degrees of freedom which have been created before the latest operation of the creation field operator, while the dotted lines indicate those created by the last operation. The arrows indicate the operation of creation (from left to right) and annihilation (from right to left) of D-particles.

The presence of the dummy components, the feature (a) above, is taken into account by assuming a set of special projection conditions, such as

$$\partial_{y_1} \phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle = 0, \quad \partial_{z_k} \phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle = 0 \quad \text{for } k \geq 2,$$

$$\partial_{y_2} \phi^+[z^{(2)}, \bar{z}^{(2)}]\phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle = 0, \quad \partial_{z_k} \phi^+[z^{(2)}, \bar{z}^{(2)}]\phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle = 0 \quad \text{for } k \geq 3.$$

The feature (b), a continuous quantum statistics corresponding to gauge invariance, is taken into account by assuming symmetry constraints such as

$$\begin{aligned} \phi^+[(UXU^{-1})_{12}, (UXU^{-1})_{21}, (UXU^{-1})_{22}]\phi^+[(UXU^{-1})_{11}]|0\rangle \\ = \phi^+[z^{(2)}, \bar{z}^{(2)}]\phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle \end{aligned}$$

as a natural extension of (5.2). The action of the annihilation operator is defined as

$$\begin{aligned} \phi^-[z, \bar{z}]|0\rangle &= 0, \\ \phi^-[z, \bar{z}]\phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle &= \delta(x_1 - x_1^{(1)})\delta(y_1) \prod_{k \geq 2} \delta(z_k)|0\rangle, \\ \phi^-[z, \bar{z}]\phi^+[z^{(2)}, \bar{z}^{(2)}]\phi^+[z^{(1)}, \bar{z}^{(1)}]|0\rangle \\ &= \int dU \delta^2(z_1 - (UXU^{-1})_{12})\delta(x_2 - (UXU^{-1})_{22}) \left( \prod_{k \geq 3} \delta(z_k) \right) \phi^+[(UXU^{-1})_{11}]|0\rangle \end{aligned}$$

which are again natural extensions of the ordinary one (5.3) corresponding to the usual discrete statistical symmetry. Actually, the algebra of these field operators

has various peculiar features such as non-associativity, and hence we need more sophisticated notations to treat this system precisely.

With this apparatus, it is possible to represent all possible gauge invariants in terms of bilinear operators of the form

$$\langle \phi^+, F\phi^- \rangle \equiv \int [d^{2d}z] \phi^+[z, \bar{z}] F\phi^-[z, \bar{z}] \quad (5.5)$$

where  $F$  is an appropriate operator acting on arbitrary functions on the infinite-dimensional coordinate space of  $(z_n, \bar{z}_n)$ . For instance, the Schrödinger equation takes the form  $\mathcal{H}|\Psi\rangle = 0$ , where

$$\begin{aligned} \mathcal{H} = & i(4\langle \phi^+, \phi^- \rangle + 1)\partial_t + \\ & 2g_s\ell_s \left( (\langle \phi^+, \phi^- \rangle + 1)\langle \phi^+, \partial_{\bar{z}^i} \cdot \partial_{z^i} \phi^- \rangle + 3\langle \phi^+, \partial_{\bar{z}^i} \phi^- \rangle \cdot \langle \phi^+, \partial_{z^i} \phi^- \rangle \right) \\ & + \frac{1}{2g_s\ell_s^5} (4\langle \phi^+, \phi^- \rangle + 1)(\langle \phi^+, \phi^- \rangle + 1)\langle \phi^+, \left( (\bar{z}^i \cdot z^j)^2 - (\bar{z}^i \cdot z^j)(\bar{z}^j \cdot z^i) \right) \phi^- \rangle \phi^- \rangle. \end{aligned} \quad (5.6)$$

In the large  $N$  limit and in the center-of-mass frame satisfying

$$\langle \phi^+, \partial_{z^i} \phi^- \rangle = 0,$$

this is simplified to

$$i\partial_t|\Psi\rangle = \left[ -\frac{g_s\ell_s}{2} \langle \phi^+, \partial_{\bar{z}^i} \cdot \partial_{z^i} \phi^- \rangle - \frac{\langle \phi^+, \phi^- \rangle}{2g_s\ell_s^5} \langle \phi^+, \left( (\bar{z}^i \cdot z^j)^2 - (\bar{z}^i \cdot z^j)(\bar{z}^j \cdot z^i) \right) \phi^- \rangle \phi^- \right] |\Psi\rangle. \quad (5.7)$$

These dynamical equations are of course consistent with the scaling symmetry which characterizes the space-time uncertainty relation,

$$(z^i, \bar{z}^i) \rightarrow \lambda(z^i, \bar{z}^i) \quad t \rightarrow \lambda^{-1}t, \quad g_s \rightarrow \lambda^3 g_s$$

providing that the field operators have scaling property such that the combinations  $\sqrt{[d^{2d}z]} \phi^\pm[z, \bar{z}]$  have zero scaling dimension.

## §6. Conclusion: a tribute to Yukawa (1907-1981)

Finally, I would like to give a brief discussion on the nature of our D-particle field, in connection with the old idea of ‘elementary domains’ by Yukawa. It is well known that Yukawa devoted his research life after the meson theory mainly to attempts toward various non-local field theories, in hopes of overcoming the ultraviolet difficulty. The idea of elementary domains constituted his last work<sup>20)</sup> along this direction. He proposed to introduce a quantized field  $\Psi(D, t)$  which is defined on some extended spatial domains  $D$ . He thought that various excitations with respect to the domain  $D$  would give different elementary particles. Of course, this is basically the way that we interpret various particle states in string theory. In this sense, a general string field  $\Psi[x(\sigma)]$  with the domain being the one-dimensional string region is quite

reminiscent of the Yukawa field, if we ignore a difference that his commutation relation between creation and annihilation operators deviates from the canonical one in some important point. In contrast to this, the string field at least in the light-cone gauge obeys the usual canonical commutators with respect to the loop space of  $x(\sigma)$ . It seems also that the idea could encompass<sup>21)22)</sup> even the graviton and hence General Relativity would not have been within his imagination, since he has never discussed gravity from his viewpoint of non-local field theories, at least to my knowledge.

According to his own account, Yukawa's proposal was inspired by the famous words by a great Chinese poet Li Po (701-762), saying<sup>\*)</sup>

*“Heaven and earth are the inn for all things,  
the light and shadow the traveler of a hundred generations.”*

Yukawa described the inspiration coming to his mind as<sup>23)</sup> *“If we replace the words ‘heaven and earth’ by the whole 3 dimensional space, and ‘all things’ by the elementary particles, the 3 dimensional space must consist of indivisible minimal volumes, and the elementary particles must occupy some of them. Such a minimal volume may be called an elementary domain. ...”*.

The base space of our D-particle field theory also has some resemblance with the idea of elementary domains. In our case, the degrees of freedom representing the domains is expressed as the infinite dimensional complex vector space of  $\{z_n, \bar{z}_n\} \sim D$ . Its nature is characterized as follows:

- The elementary domains of D-brane fields are represented by open-string degrees of freedom and as such are ‘fuzzier’ than Yukawa’s. Our elementary domains are a sort of “clouds of threads” emanating from D-particles.

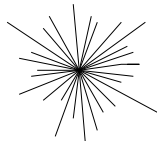


Fig. 3. A D-particle domain as a cloud of open strings.

- The D-brane fields do not satisfy canonical commutation relations and are instead characterized by an entirely new ‘continuous’ quantum statistics. The domains are mutually permeable and osmotic, in contrast to Yukawa’s tighter *indivisible* domains.
- Our domains are much more dynamical. Infinite components of the base-space coordinates are actually latent, and the complexity of domains depends on the number of D-particles. The length scale of the domain is governed by the space-time uncertainty relation.
- The theory includes gravity through open/closed string duality in the low-

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<sup>\*)</sup> This is in the preface to “Holding a Banquet in the Peach and Pear on a Spring Night”. The translation adopted in the text here is due to *Stonebridge Press* (<http://www.stonebridge.com/>) who discusses the quotations of the same sentences made by Basho Matsuo (1644-1694), the greatest ‘Haiku’ poet in the Edo period in Japan.

energy limit. Even in the non-relativistic approximation, the non-linear general-relativistic interactions<sup>24)</sup> of D-particle can emerge as a quantum loop effect of the Yang-Mills interaction embodied in the field-theory Hamiltonian. So ‘*all things*’ could include even space-time geometry in the bulk.

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